Ground Flash Fraction Retrieval Algorithm

GLM Science Meeting
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photo by Chuck Doswell

Desired Future Application

 # GLM Flashes → **Ground Flash Fraction Retrieval Algorithm** (today's talk)

↓

 # GLM Ground Flashes | **# GLM Cloud Flashes**

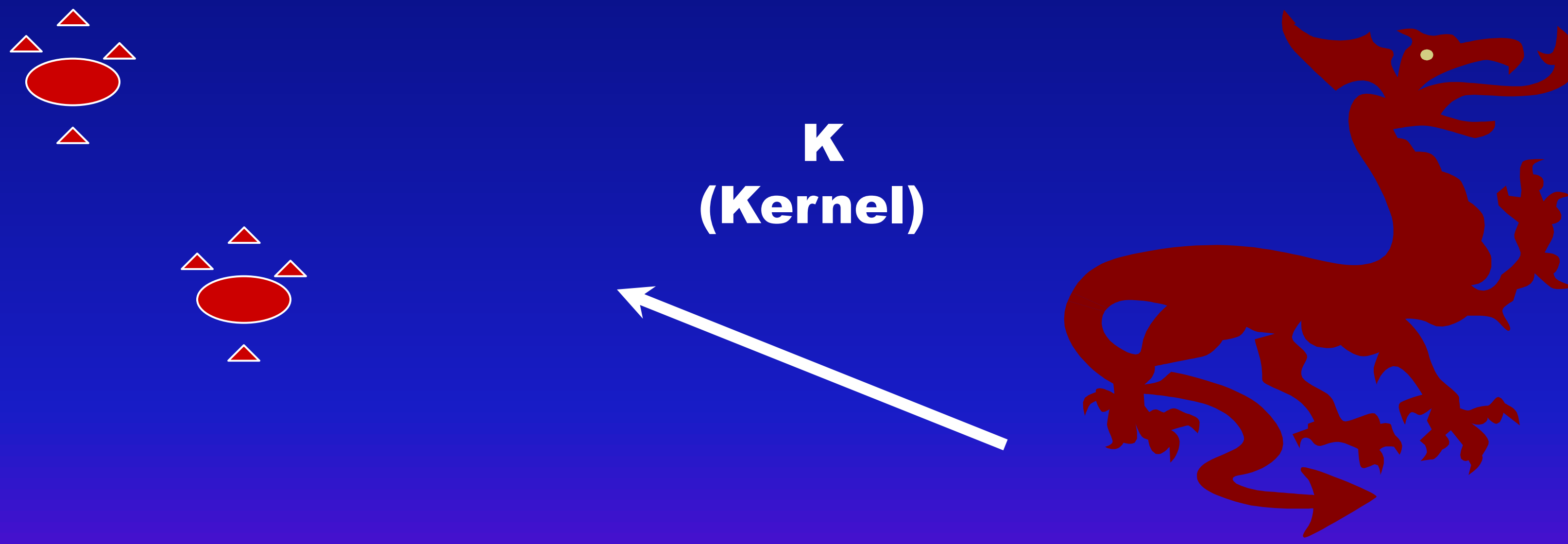
← **MSFC LNOM (lightning NOx)**

↓

Air Quality Models (e.g. CMAQ)
Global Chemistry/Climate Models
(e.g. GISS Model E, Geos Chem)

Algorithm is used to solve an Inversion Problem

Measurements $\vec{g}$ **Unknown** $\vec{f}$



(Footprints) **(Dragon)**

What % Strike Ground?

Consider a (linear) Optics Analog

Mueller Matrix: A matrix which can be used to reproduce the effect of a given optical element when applied to a Stokes vector.

Optical Element	Mueller Matrix (Singular)
Linear horizontal polarizer	$\frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
Right-handed circular polarizer	$\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$

In Reality:

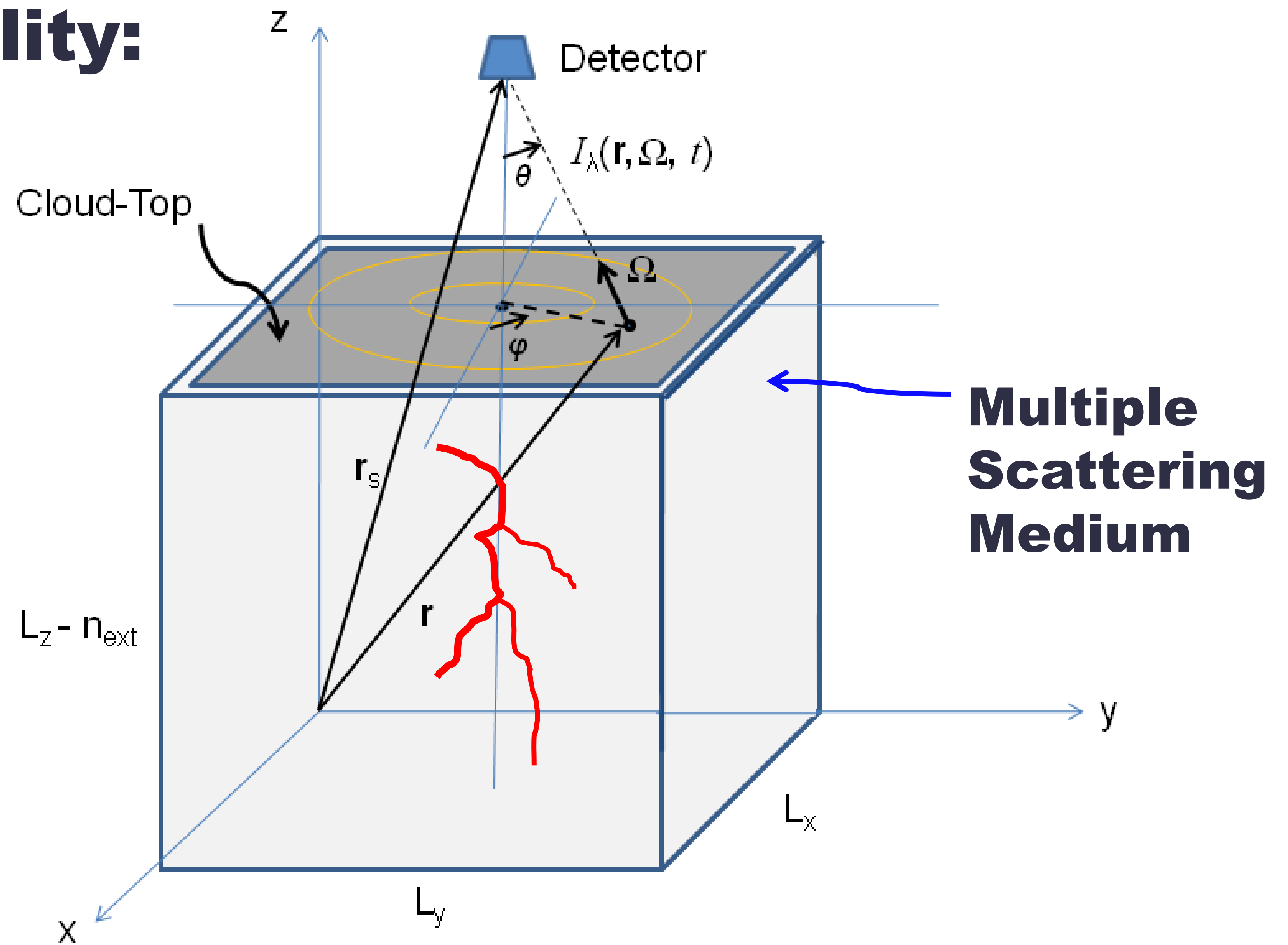


Figure 3. Geometry involved with computing the radiant energy received by a detector from the diffuse, transient lightning cloud-top emission. Note that the cloud is embedded within the solution space volume $V = L_x L_y L_z$ and the top vertical layer of the solution space, having thickness n_{ext}, has been removed for clarity. The point shown on cloud-top is $\mathbf{r}$ = (x, y, h), and the detector is located at $\mathbf{r}_s$ = (x_s, y_s, z_s).

Actual Forward Multiple Scattering Problem

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Diffusion model for lightning radiative transfer

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Abstract. A one-speed Boltzmann transport theory, with diffusion approximations, is applied to study the radiative transfer properties of lightning in optically thick thunderclouds. Near-infrared ($\lambda = 0.7774$ μm) photons associated with a prominent oxygen emission triplet in the lighting spectrum are considered. Transient and spatially complex lightning radiation sources are placed inside a rectangular parallelepiped thundercloud geometry and the effects of multiple scattering are studied. The cloud is assumed to be composed of a homogeneous collection of identical spherical water droplets, each droplet a nearly conservative, anisotropic scatterer. Conceptually, we treat the thundercloud like a nuclear reactor, with photons replaced by neutrons, and utilize standard one-speed neutron diffusion techniques common in nuclear reactor analyses. Valid analytic results for the intensity distribution (expanded in spherical harmonics) are obtained for regions sufficiently far from sources. Model estimates of the arrival-time delay and pulse width broadening of lightning signals radiated from within the cloud are determined and the results are in good agreement with both experimental data and previous Monte Carlo estimates. Additional model studies of this kind will be used to study the general information content of cloud top lightning radiation signatures.

Introduction

Satellite observations of lightning have been gathered for several years [*Sparrow and Ney*, 1968; *Vorphal et al.*, 1970; *Sparrow and Ney*, 1971; *Turman*, 1978; *Orville and Henderson*, 1986] and the requirements for a comprehensive space-based lightning detection system has been addressed by *Christensen et al.* [1980]. These requirements have been more clearly defined in subsequent studies [*Few et al.*, 1980; *Davis et al.*, 1983] and the details of a proposed lightning mapper system (LMS) are described by *Christian et al.* [1989]. The LMS would detect and locate lightning from geosynchronous orbit, allowing for a broader areal coverage of world-wide lightning and more continuous thunderstorm monitoring. Similar benefits will be gained by a related, low Earth orbiting lightning imaging sensor (LIS) [*Christian et al.*, 1992]. Recently, it has been shown that the lightning flashing rate is an indicator of total rain volume [*Goodman and MacGorman*, 1986; *Goodman and Buechler*, 1990] and the total Maxwell current density above thunderstorms [*Blakeslee et al.*, 1989]. This suggests that staring lightning imagers might be used to remotely infer a broad range of meteorological/electrical phenomena, however, the usefulness of other types of space-based observations of lightning, including spectral, temporal, and polarization measurements has yet to be studied in great detail.

In order to fully benefit from spaceborne observations of lightning it is desirable to investigate the general information content of cloud top lightning signals radiated to space and the patterns of cloud top illumination due to lightning. Lightning optical waveform data obtained from high-altitude U2 missions above thunderstorms have been collected and examined in recent years [*Christian et al.*, 1983; *Christian and Goodman*, 1987; *Goodman, et al.*, 1988], however, only one (Monte Carlo) study due to *Thomason and Krider* [1982] is available to interpret these data. In this paper we compliment the study of *Thomason and Krider* [1982] by describing a deterministic (i.e., nonstochastic) model for simulating the transfer of lightning produced photons in optically thick thunderclouds. In accordance with the narrow-band filter design of the LIS we consider photons that are associated with a prominent oxygen emission triplet centered near $\lambda \approx 0.7774$ μm in the lightning spectrum.

Studies related to the transfer of solar and terrestrial radiation in clouds are extensive in the atmospheric sciences (see for instance, *Bauer* [1964], *Deirmendjian* [1964], *Twomey et al.* [1967], *Plass and Kattawar* [1968] *Danielson et al.* [1969], and *Hansen* [1969, 1971]) and various theoretical approaches are summarized in *Lenoble* [1977]. Unfortunately, these investigations typically deal with the steady state reflection, transmission, and absorption of radiation in semi-infinite plane parallel clouds with external plane parallel cloud illumination. Such assumptions are crude or inadequate in the case of lightning. Radiant energy emitted from a lightning channel is highly transient and horizontally inhomogeneous particularly for very tortuous channels. In addition, the (finite) thundercloud is primarily illuminated from the inside.

From a geometric standpoint, models used to determine microwave emissions from finite, three-dimensional hydrometeor clouds [*Weinman and Davies*, 1978; *Kummerow and Weinman*, 1988] offer one possible analog to the problem considered here. However, the physical properties of photon extinction differ at these frequencies and droplet sizes and nontransient microwave source emissions were considered. Those studies that have explicitly dealt with visible or infrared radiative transfer in finite cloud geometries [*Uesugi and Tsujita*, 1969; *Busygin et al.*, 1973; *McKee and Cox*, 1974,

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Eqs. (21) is the Forward Problem … it's ugly.

Inverse Problem is uglier.

Note that (18) is quite general, allowing one to specify an arbitrary isotropic, transient lightning source $\rho_Q(\mathbf{r},t)$ inside an arbitrary convex cloud volume. The lightning source function, $\rho_Q(\mathbf{r},t)$, can have discrete or continuous dependence on the space and time variables. If the time derivative and various integral projections implied by (18) can be performed analytically (quite possible for a vast array of source functions), and if (16) can be solved for the eigenvalues and eigenfunctions of the cloud geometry in question (this is possible for spherical, block, or cylindrical geometries), one can use (7), and the definition for $\mathbf{J}$ in (8) to reconstruct an analytic form for the intensity distribution $I(\mathbf{r},\Omega,t)$ anywhere in the volume bounded by S_{ext}.

Homogeneous Rectangular Pararallelepiped Cloud

As alluded to in the previous section, a variety of lightning sources and cloud geometries/properties are possible when using the solution given in (18). In this writing we consider one or more transient point sources embedded within a homogeneous (D = const) rectangular parallelepiped cloud geometry with Cartesian dimensions (d_x, d_y, d_z). The point sources are allowed to turn on and off simultaneously and/or in arbitrary succession. Letting $t = 0$ correspond to a time prior to any lightning event and letting the weighting factor, w_j, represent the photon strength at location $\mathbf{r}_j$ and time t_j, we may write the source derivative as

$$\frac{\partial \rho_Q(\mathbf{r},t')}{\partial t'} = \sum_{j=1}^{N} w_j \delta(\mathbf{r}-\mathbf{r}_j)\delta(t'-t_j), \qquad (19)$$

where N is the total number of point sources composing the lightning source. Solving (16) for a parallelepiped cloud geometry, the eigenfunctions and corresponding eigenvalues are:

$$\psi_{lmn}(\mathbf{r}) = \sin\left(\frac{\pi l x}{d_x}\right)\sin\left(\frac{\pi m y}{d_y}\right)\sin\left(\frac{\pi n z}{d_z}\right),$$

$$\lambda_{lmn}^2 = D\pi^2\left[(l/d_x)^2+(m/d_y)^2+(n/d_z)^2\right]+(1-\omega_o)Kc, \qquad (20)$$

where the i subscript in (16) is necessarily replaced by the 3-tuple (l,m,n). These three indices each range between 1 and infinity. Carrying out the operations in (18) and using (7) and (8), we obtain the intensity distribution inside V as

$$I(\mathbf{r},\Omega,t) = \frac{2}{\pi V}\sum_{l=1}^{\infty}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left[\psi_{lmn}(\mathbf{r}) - \frac{3D}{c}\Omega\cdot\nabla\psi_{lmn}(\mathbf{r})\right] \times \sum_{j=1}^{N} w_j \psi_{lmn}(\mathbf{r}_j) e^{-\lambda_{lmn}^2 (t-t_j)}. \qquad (21)$$

In practice, we estimate the triple infinite sum in (21) by truncating it at sufficiently large values of l, m, and n.

Model Results and Discussion

In this investigation we stress the reasonability of our results by choosing cases that are somewhat intuitive. We begin by selecting a cubic cloud ($d_x = d_y = d_z$ = 10 km) and a single-

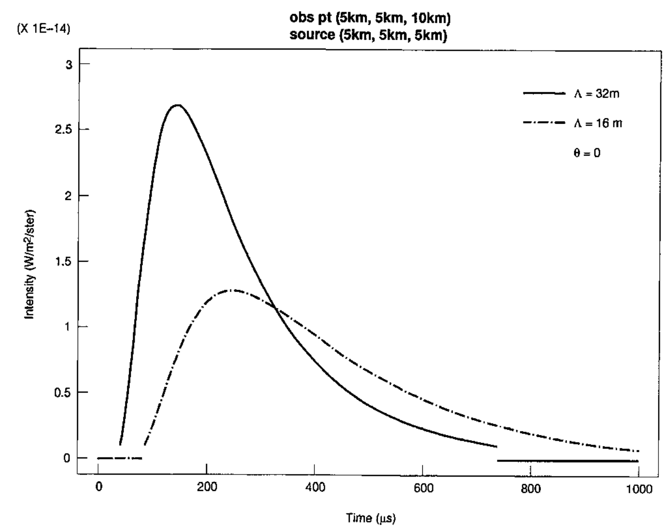
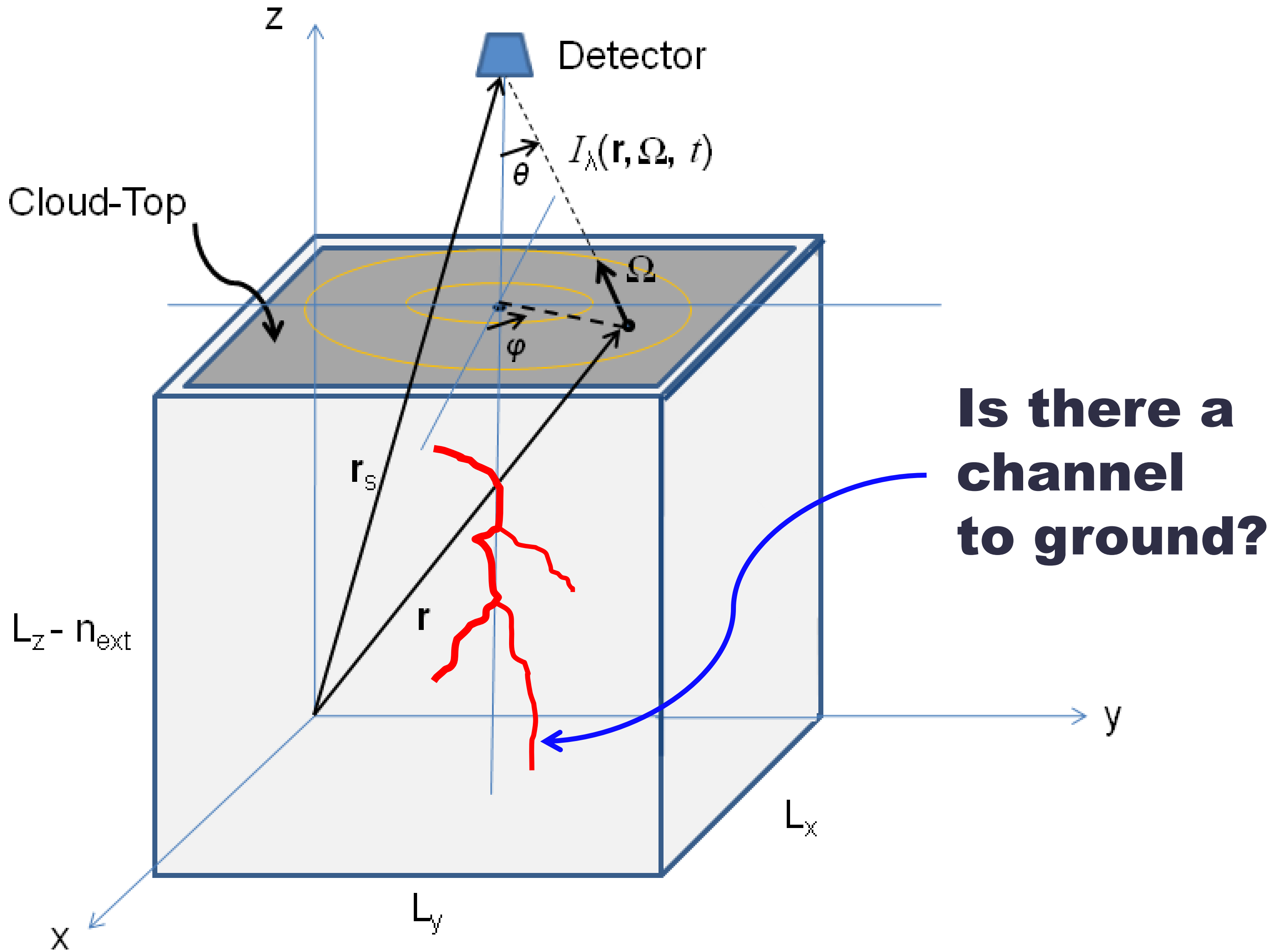


Figure 4. Effect of cloud optical thickness on a transient point source located at the center of a cubic cloud (dimension = 10 km). Detector is at or above the center of cloud top and is looking down. Note that the "thicker" cloud (smaller mean free path) time delays and time broadens the source more, as expected. The sharp breaks in the intensity curve at about 100 and 750 μs indicate when acceptable truncation error thresholds are met.



Formal Inverse Problem:

Reconstruct Channel from Intensity

Historical Recap

- Several studied the forward problem
  - ✓Thomason & Krider 1982 (Monte Carlo)
  - ✓Koshak et al. 1994 (Boltzmann Diffusion)
  - ✓*Suszcynsky et al. 2000 (optical & vhf data, not theory)*
  - ✓Light et al. 2001 (Monte Carlo)
  - ✓Davis & Marshak 2002 (Green's functions)

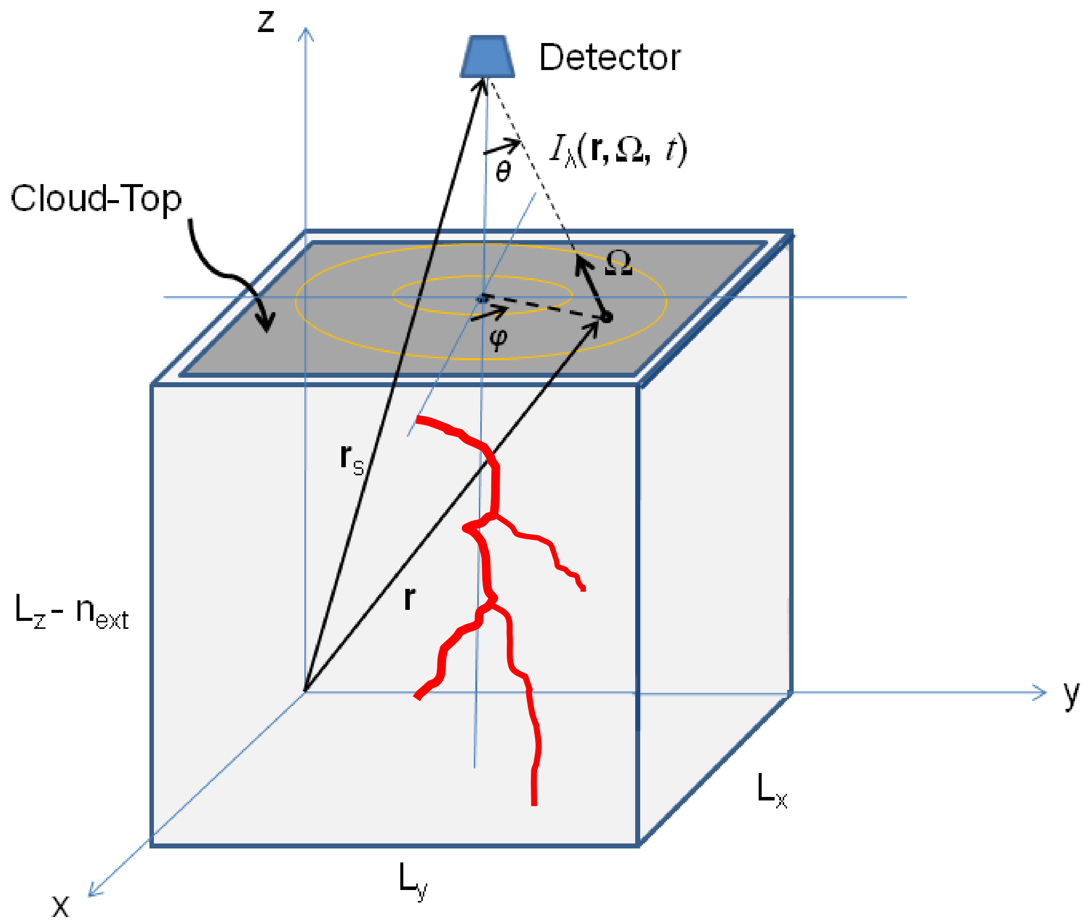
- Nobody published the inverse problem for channel reconstruction; flash-by-flash-discrimination (FBFD)

- Neural Net (Boccippio … unknown status); probabilistic FBFD

- Bayesian Inversion (Koshak); ground flash fraction retrieval

Bayesian Inversion

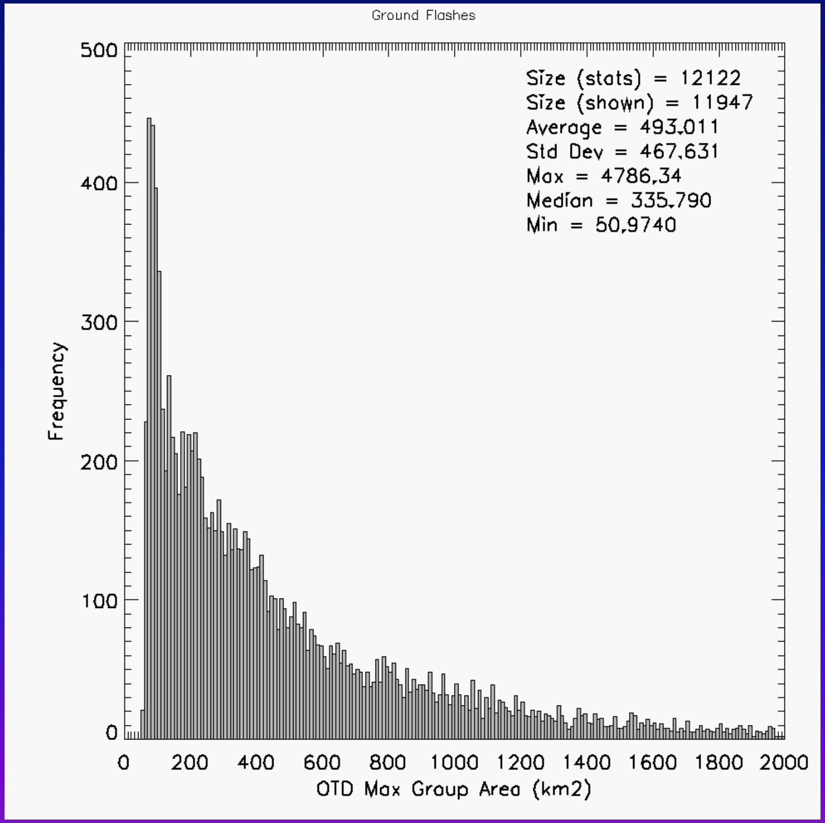
Use just 1 physical parameter



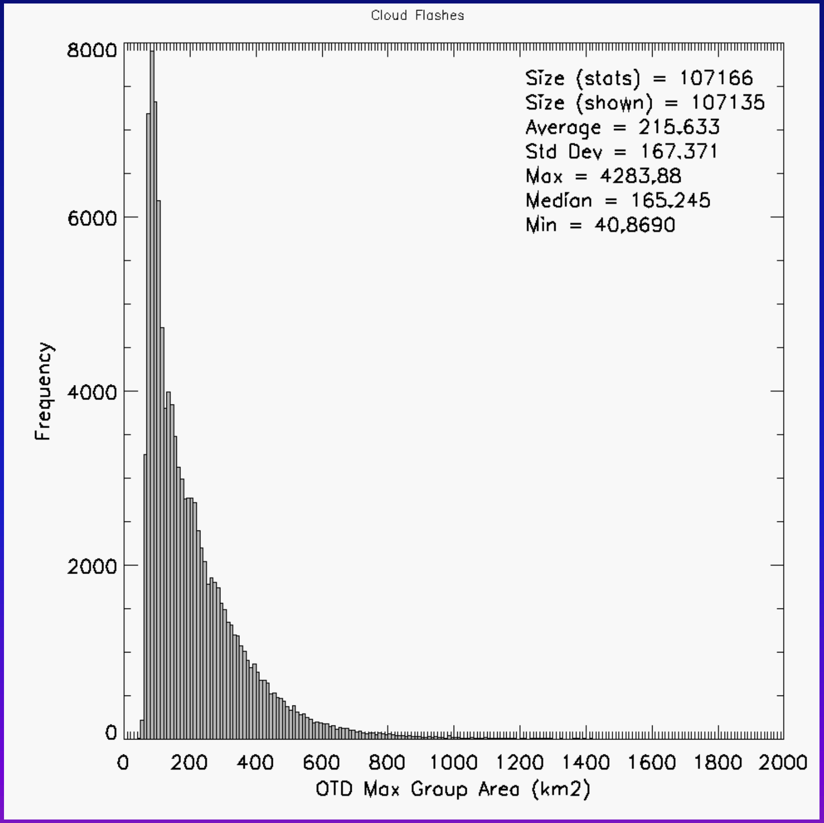
Find & examine the "group" in the flash having the Maximum Area.

Distributions of the Maximum Group Area (MGA)

Ground Flashes

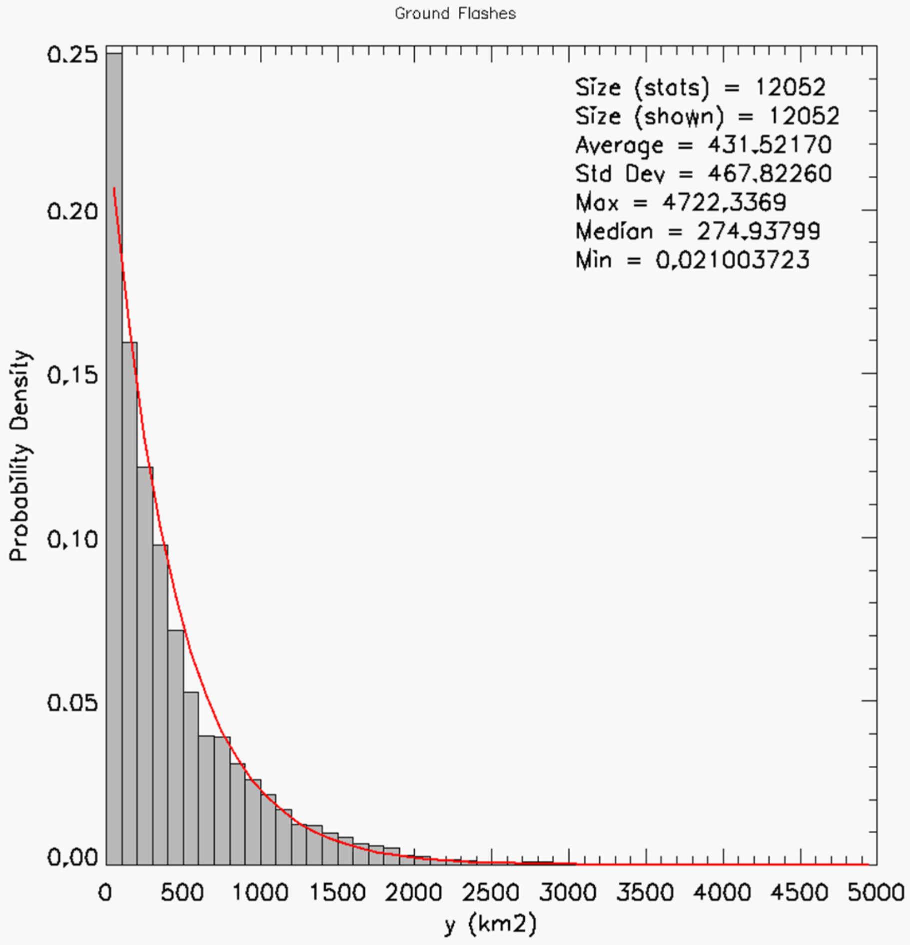


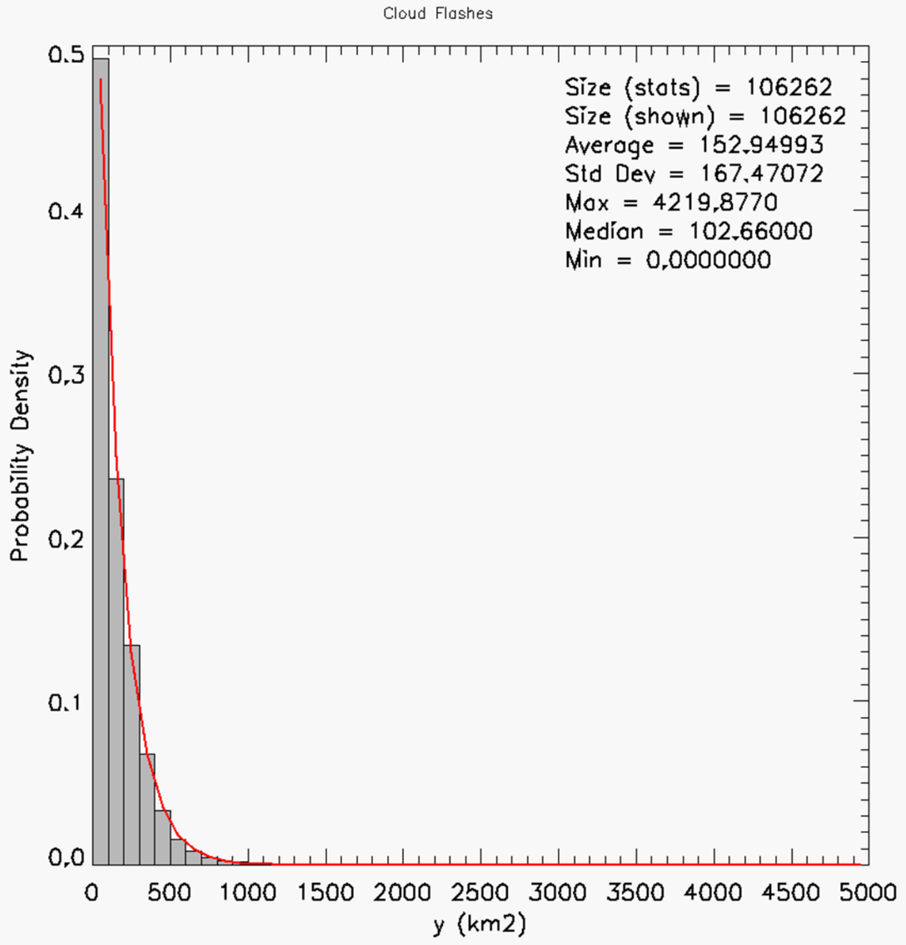
Cloud Flashes



Shifted MGA

$$y = \text{MGA} - 64\ \text{km}^2$$





Mixed Exponential Distribution Model

Basic Definitions:

$\boldsymbol{x} =$ Maximum Group Area (MGA) in a flash

$\boldsymbol{y} = \boldsymbol{x} - 64\boldsymbol{km}^2$ (Shifted MGA)

$N_g =$ # Ground flashes in lat/lon bin of interest

$N =$ # Flashes in lat/lon bin of interest

$\alpha = N_g / N$ (Ground flash fraction)

$\mu_g =$ Population mean $\boldsymbol{y}$ for ground flashes (in lat/lon bin of interest)

$\mu_c =$ Population mean $\boldsymbol{y}$ for cloud flashes (in lat/lon bin of interest)

Distribution of MGA modeled as a <u>Mixed</u> Exponential Distribution:

$$\boxed{p(y) = \alpha p_g(y) + (1-\alpha) p_c(y) = \frac{\alpha}{\mu_g} e^{-y/\mu_g} + \frac{(1-\alpha)}{\mu_c} e^{-y/\mu_c}, \quad y \geq 0} \quad .$$

Population Means of y:

$$\mu_g \equiv \int_0^\infty y p_g(y) dy, \quad \mu_c \equiv \int_0^\infty y p_c(y) dy \quad .$$

Require that:

$$\mu_g > \mu_c$$

Bayesian Inversion

Bayes' Law:

$$P(\alpha,\mu_g,\mu_c\,|\,\mathbf{y})=\frac{P(\mathbf{y}\,|\,\alpha,\mu_g,\mu_c)P(\alpha,\mu_g,\mu_c)}{P(\mathbf{y})}\quad ,$$

Find parameters $\mathbf{v}=(\alpha,\mu_g,\mu_c)$ that maximize the probability on LHS.

This means one maximizes the following :

$$S(\mathbf{v})\equiv\ln\left[P(\mathbf{y}\,|\,\mathbf{v})P(\mathbf{v})\right]=\ln\prod_{i=1}^{m}p(y_i\,|\,\mathbf{v})+\ln P(\mathbf{v})=\sum_{i=1}^{m}\ln\left[\frac{\alpha}{\mu_g}e^{-y_i/\mu_g}+\frac{(1-\alpha)}{\mu_c}e^{-y_i/\mu_c}\right]+\ln P(\mathbf{v})\,,$$

Formally :

$$\frac{\partial S(\mathbf{v})}{\partial\mathbf{v}}=\mathbf{0}\quad\Rightarrow\quad\mathbf{v}=\text{"Maximum A Posteriori (MAP) Solution"}$$

Practically :

Use Broyden-Fletcher-Goldfarb-Shannon variant of Davidon-Fletcher-Powell numerical method to minimize $-S(\mathbf{v})$. Also, $P(\mathbf{v})$ is simplified by assuming model parameter independence, with $P(\alpha)$ uniform, and $P(\mu_g)$ & $P(\mu_c)$ both normal distributions.

Initialization for Numerical Search

Population mean and variance of the mixture:

$$\mu \equiv \int_{-\infty}^{\infty} y p(y) dy = \alpha\mu_g + (1-\alpha)\mu_c \quad (\mathit{line}),$$

$$\sigma^2 \equiv \int_{-\infty}^{\infty} (y-\mu)^2 p(y) dy = \alpha(2-\alpha)\mu_g^2 - 2\alpha(1-\alpha)\mu_g\mu_c + (1-\alpha^2)\mu_c^2 \quad (\mathit{rotated\ ellipse}).$$

Using 1st equation to solve for μ_c gives an equation quadratic in μ_g , hence:

$$\mu_g = \mu + \sqrt{\frac{1}{2}\left(\frac{1-\alpha}{\alpha}\right)\left(\sigma^2 - \mu^2\right)}$$

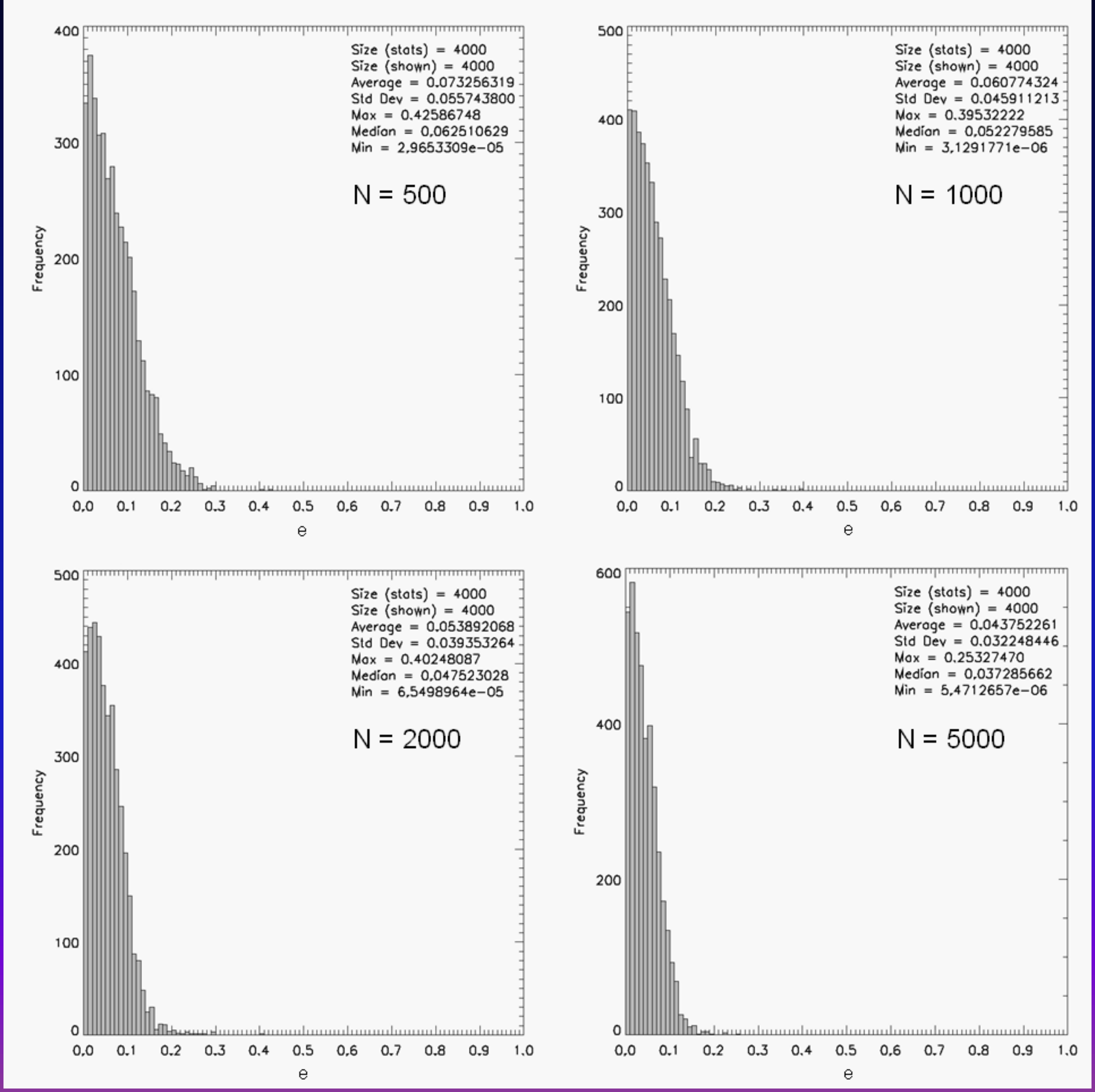
$$\mu_c = \mu - \sqrt{\frac{1}{2}\left(\frac{\alpha}{1-\alpha}\right)\left(\sigma^2 - \mu^2\right)}$$

where initialization as follows results in an initialization of (μ_g, μ_c) :

$$\alpha = 0.5, \quad \mu = \bar{y}, \quad \sigma = s_y$$

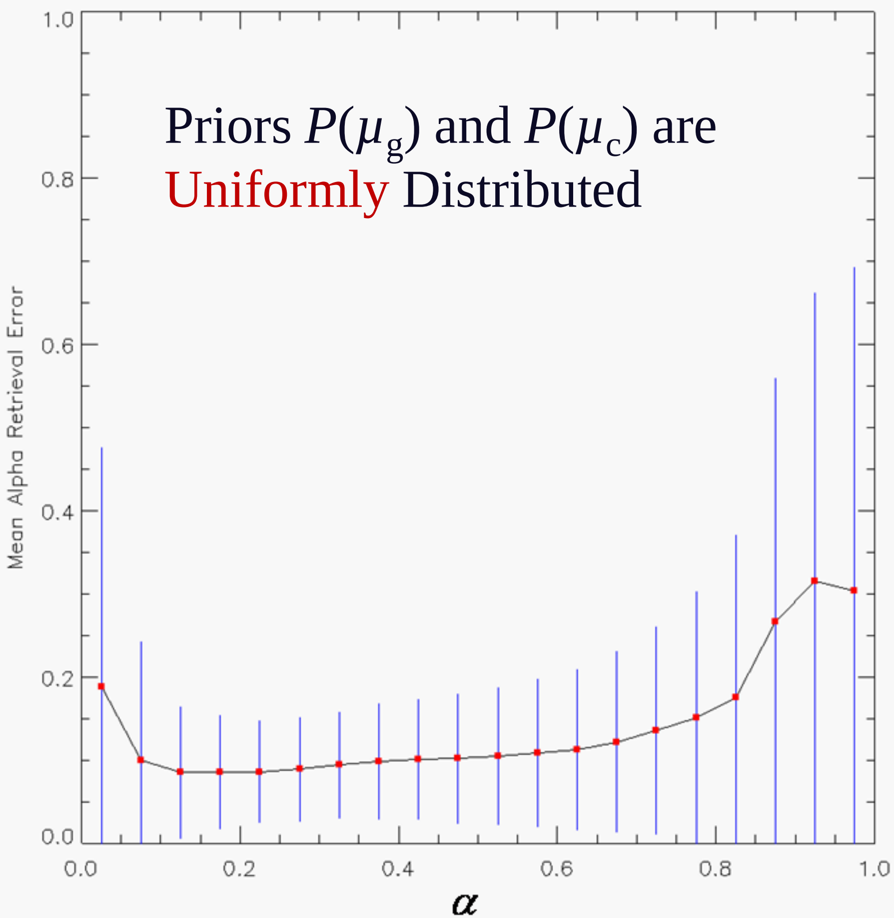
Retrieval Errors with Increasing N

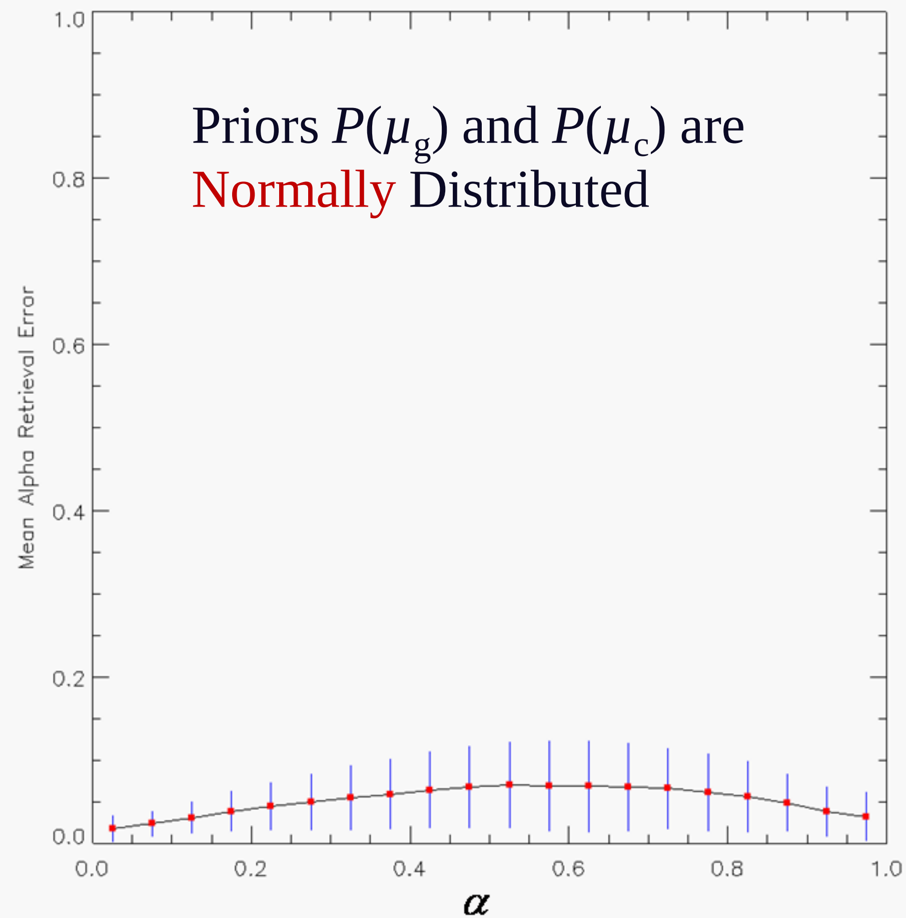
(N = # flashes observed)



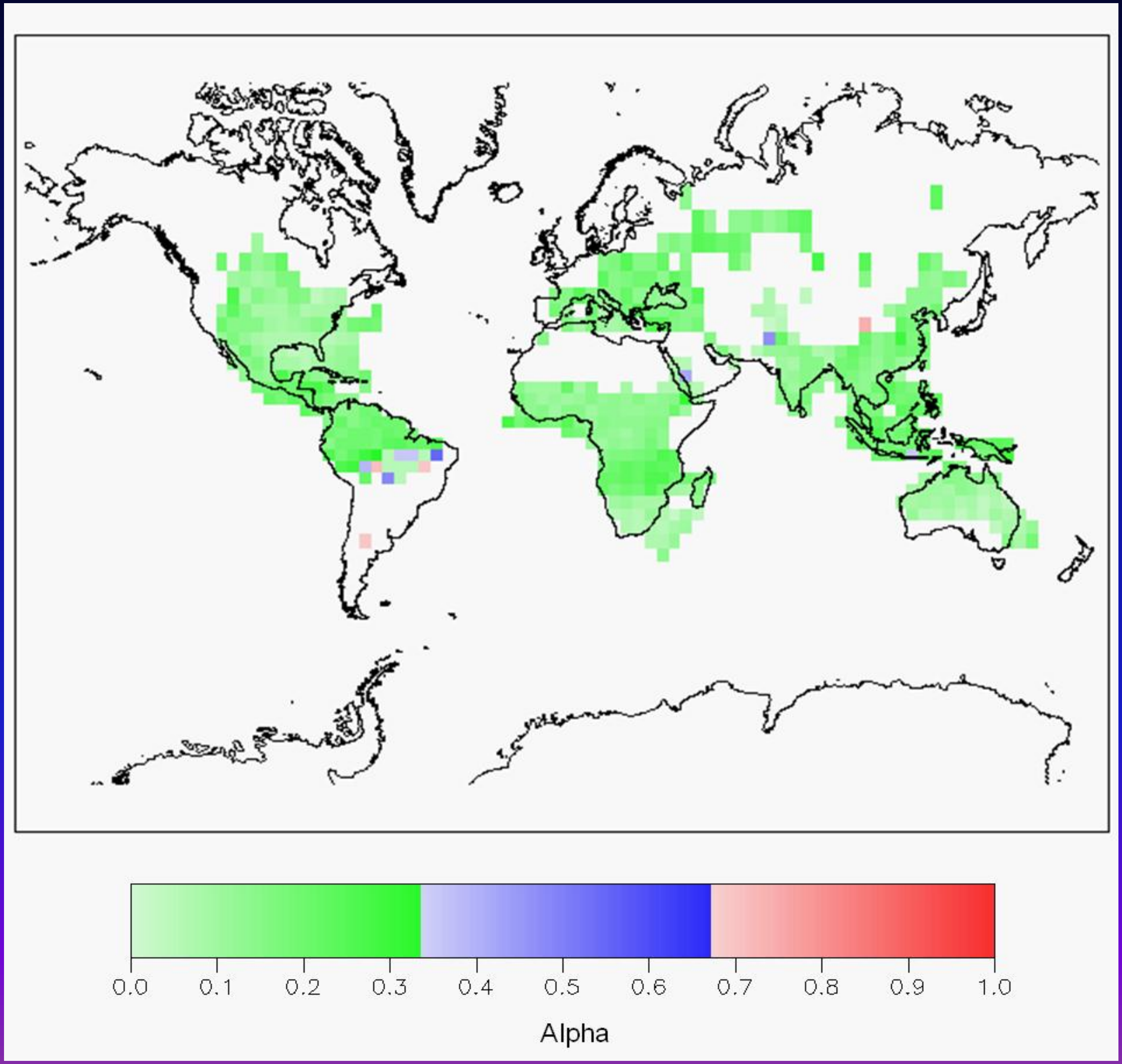
Improvements using Normal Priors

$$P(\alpha, \mu_g, \mu_c) = P(\alpha)\, P(\mu_g)\, P(\mu_c) = 1 \cdot P(\mu_g)\, P(\mu_c)$$





Retrieved Ground Flash Fraction



For More Details …

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